PPML and its Comonadic Semantics

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Motivation: Data-aware Logics

- *Data-aware logics* reason over structures where each point stores *data values*, e.g. *data trees*
- $\text{XPath}_=(\downarrow_+)$ is a fragment of the XML query language XPath which
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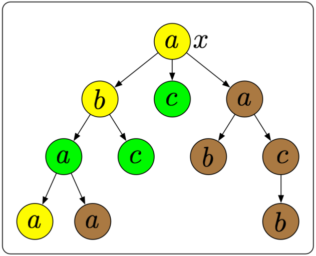

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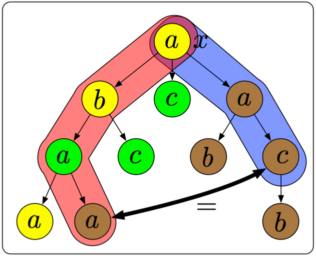

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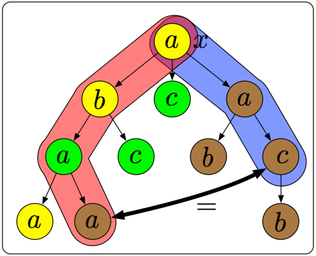

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DataGL [Baelde, Lunel, and Schmitz 2016] is the fragment where allowed expressions are of the form

$$\langle \varepsilon = \downarrow_+[\psi] \rangle \quad \text{or} \quad \langle \varepsilon \neq \downarrow_+[\psi] \rangle$$

Path Predicate Modal Logic (PPML)

PPML arises as a natural extension of Basic Modal Logic: instead of propositional letters we consider symbols of arbitrary (finite) arity $R_1, \dots R_m$:

$$\varphi ::= \varphi \wedge \psi \mid \varphi \vee \psi \mid \neg\varphi \mid \Diamond\varphi \mid R_i \quad (i \in \{1, \dots, m\})$$

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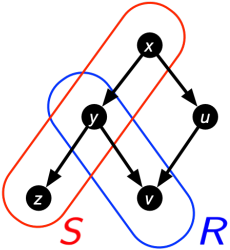

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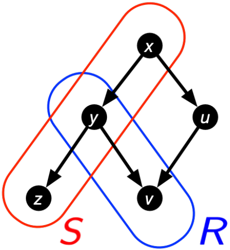

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DataGL can be recovered as a semantic restriction of PPML using $\sigma_{DGL} = \{R_0, R_=, p, q, r, \ldots\}$

PPML Bisimulation

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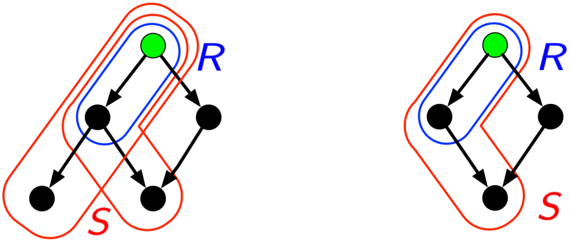

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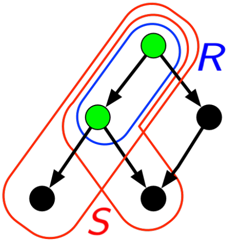

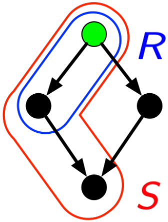

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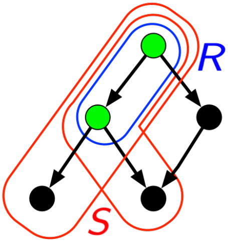

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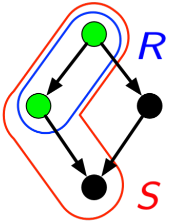

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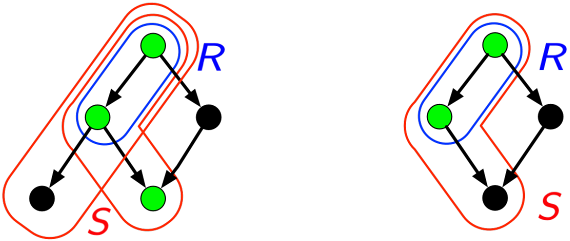

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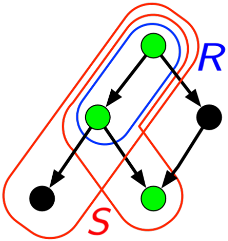

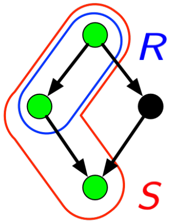

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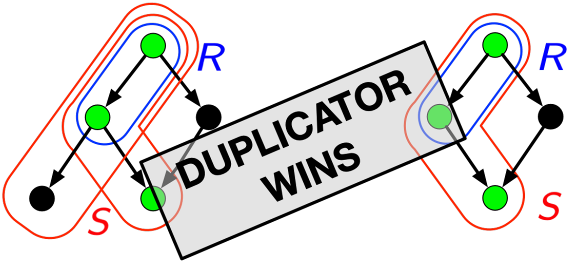

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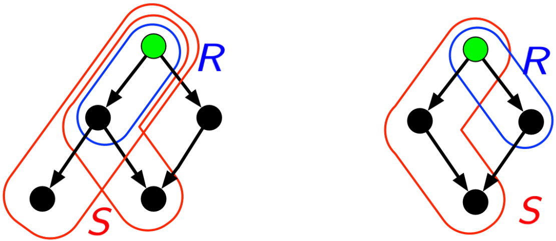

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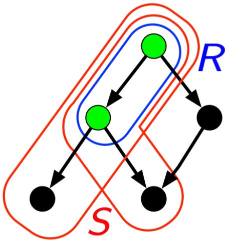

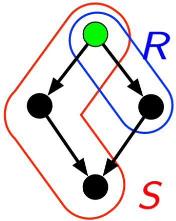

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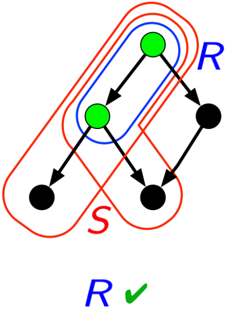

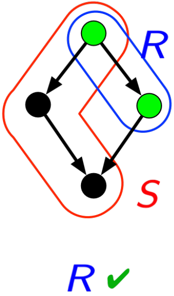

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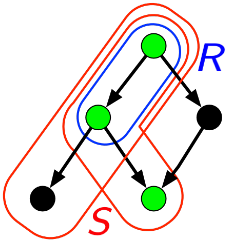

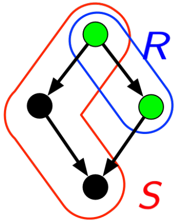

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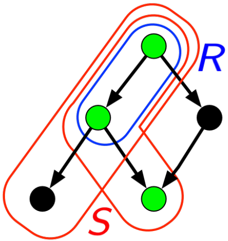

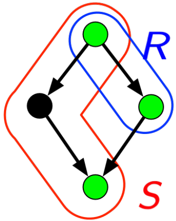

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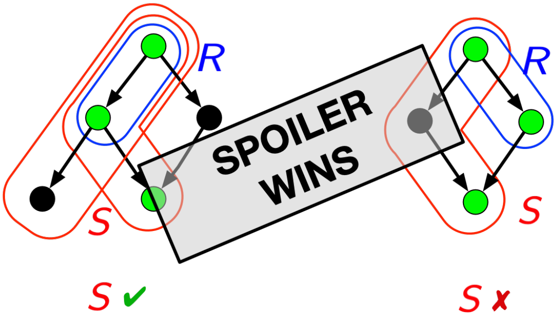

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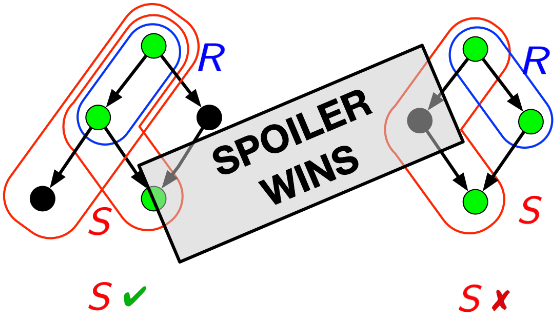

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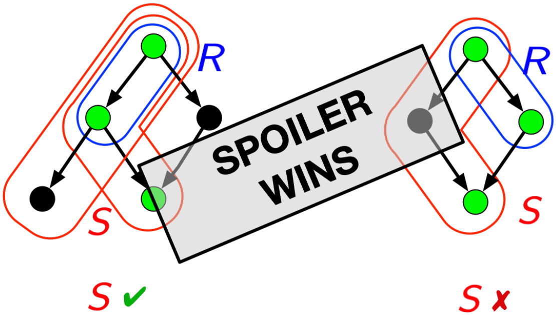

- In the bisimulation game, Spoiler can also switch between models.
- As is to be expected, these games characterize logical equivalences $\equiv_k$ (modal depth $\leq k$ fragment), $\equiv_k^{\Diamond}$ (negation-free subfragment).

The PPML Comonad

Definition

$$\mathbb{C}_k : \mathsf{Struct}_*(\sigma) \to \mathsf{Struct}_*(\sigma) \qquad \sigma \ni R_0$$
$$(\mathcal{A}, a) \mapsto \mathbb{C}_k(\mathcal{A}, a)$$

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- Universe of $\mathbb{C}_k(\mathcal{A}, a)$ is the *unravelling* of $\mathcal{A}$ starting from a along R_0 (just like for Modal Comonad $\mathbb{M}_k$)
- $R^{\mathbb{C}_k(\mathcal{A},a)}(s_1, \ldots, s_n)$ iff
  1. each sequence in the tuple is an immediate successor of the previous one
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Proposition

$\mathbb{C}_k$ defines a subcomonad of

- the Ehrenfeucht-Fraïssé Comonad $\mathbb{E}_k$
- the Pebbling Comonad $\mathbb{P}_{N(\sigma)}$ with $N(\sigma) = \max_{R \in \sigma}(\mathrm{arity}(R))$

(both lifted to $\mathsf{Struct}_*(\sigma)$)

Coalgebras of the PPML comonad

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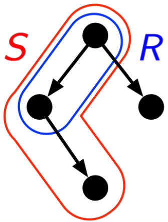

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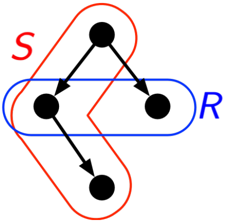

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Expressivity results

Theorem

Winning strategies for Duplicator in k-round simulation game $\mathcal{A} \to \mathcal{B}$	$\leftrightsquigarrow$	coKleisli morphisms $\mathbb{C}_k(\mathcal{A},a) \to (\mathcal{B},b)$

Thus $(\mathcal{A}, a) \equiv_k^{\Diamond} (\mathcal{B}, b)$ *iff there exist homomorphisms* $\mathbb{C}_k(\mathcal{A}, a) \to (\mathcal{B}, b)$ *and* $\mathbb{C}_k(\mathcal{B}, b) \to (\mathcal{A}, a)$.

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Theorem

$(\mathcal{A}, a) \equiv_k (\mathcal{B}, b)$ *iff there exists a* $\mathbb{C}_k$*-coalgebra* $(\mathcal{P}, p) \in \mathsf{EM}(\mathbb{C}_k)$ *and a span of* ***strong, surjective*** *homomorphisms*

$$\mathbb{C}_k(\mathcal{A}, a) \twoheadleftarrow (\mathcal{P}, p) \twoheadrightarrow \mathbb{C}_k(\mathcal{B}, b)$$

Translation into Basic Modal Logic

$$\sigma = \{R_0, R_1, \ldots, R_m\} \rightsquigarrow \widetilde{\sigma} := \{R_0, \widetilde{R}_1, \ldots, \widetilde{R}_m\}$$

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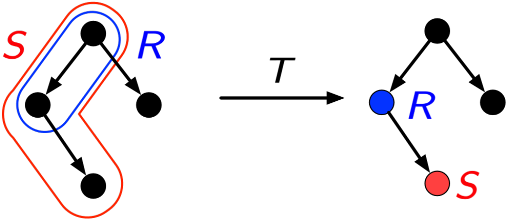
There is a functor

$$T : \mathrm{EM}(\mathbb{C}_k) \to \mathrm{EM}(\mathbb{M}_k)$$

$$(\mathbb{M}_k : \mathrm{Struct}_*(\widetilde{\sigma}) \to \mathrm{Struct}_*(\widetilde{\sigma}))$$



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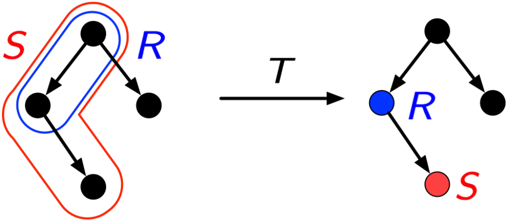

Theorem

T is fully faithful, preserves open pathwise embeddings, and its (essential) image is the full subcategory of $\mathsf{EM}(\mathbb{M}_k)$ *spanned by* $\mathbb{M}_k$*-coalgebras* $(\mathcal{A}, a)$ *such that:*

$$\text{for any } a' \in |\mathcal{A}| \text{ and } n\text{-ary relation } R \in \sigma,$$
$$\widetilde{R}^{\mathcal{A}}(a') \implies \text{the path from } a \text{ to } a' \text{ has at least } n \text{ points.}$$

Satisfiability

Lemma

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This is a first step towards the comonadic treatment of data-aware logics.

Thank you!