

①

SET THEORY

$$\exists x \exists y x \in y \in z \text{ \& } z \hookrightarrow z'$$

$$\not\Rightarrow \exists x \exists y x \in y \in z'$$

$$\text{Ex: } z = \{\{\emptyset\}\} \quad z' = \{\emptyset\}$$

TOO EXPRESSIVE!

USUAL MATHEMATICS

$$\text{IF } z \hookrightarrow z'$$

PROPERTIES/STRUCTURE OF z

$\hookrightarrow$ PROPERTIES/STRUCTURE OF z'

CATEGORY THEORY

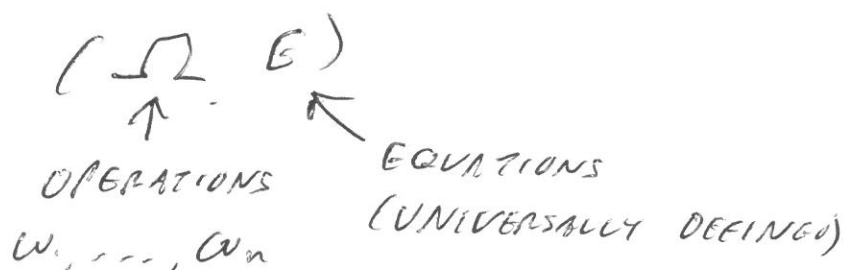
DESIGNED TO RESPECT $z \hookrightarrow z'$

Ex: TERMINAL OBJECT, PRODUCTS, LIMITS,
CARTESIAN CLOSEDNESS, ...

(NON-EX: STRICT MONOIDAL STRUCTURE.)

UNIVERSAL ALGEBRA

(E.g. GROUPS, MONOIDS, COMMUTATIVE MONOIDS, RINGS, R-MODULES, M-SETS)



(A, E)-

ALGEBRA

$(A, op_1, \dots, op_n)$

$\uparrow$
SET

$\downarrow$
 $A \times \dots \times A \xrightarrow{op_i} A$

IF $A \xrightarrow{R} X$, X ACQUIRES ALGEBRA STRUCTURE
UNIQUELY SUCH THAT R IS AN ISOMORPHISM
OF ALGEBRAS.

CATEGORY-THEORETIC FORMULATION

- ① LAWVERE THEORIES (CALLED ABSTRACT CLONES IN UNIV ALGEBRA) L
- ② FINITARY MONADS (SIMPLE DEF OF "MONAD", BUT "FINITARY" IS COMPLEX AND ESSENTIAL) T

IN BOTH FORMULATIONS, $A \xrightarrow{R} X \Rightarrow$ STRUCTURE TRANSFERS.

$(A, T_A \xrightarrow{\alpha} A)$ T -ALGEBRA $A \xrightarrow{R} X \Rightarrow X$ HAS UNIQUE T -ALG STRUCTURE S.T. R IS AN ISO OF ALGEBRAS.

(3)

WHAT ABOUT CATEGORIES WITH STRUCTURE?

- "UNIVERSAL ALGEBRA" ON CAT RATHER THAN SET
- EQUIV, LAWVERE 2-THEORIES $\mathcal{L}$
- EQUIV, 2-MONADS T ON 2-CATEGORY CAT

$(A, TA \xrightarrow{\alpha} A)$ T -ALGEBRA $A \xrightleftharpoons[R]{R} X$ ISO OF CATS

$\Rightarrow X$ HAS UNIQUE T -STR S.T. R AN ISO OF T -ALGS

BUT WHAT IF R IS AN EQUIVALENCE?

Exs: $T_t = 2$ -MONAD S.T. T_t -ALG = CAT WITH TERMINAL
OBJ

$T_{FP} = \dots$

FINITE
PRODUCTS

$T_{FL} = \dots$

$T_{MON} = \dots$

T_{MON} -ALG = MONOIDAL CAT

Non-Ex: $T_{SMON} = \dots$

T_{SMON} -ALG = STRICT MONOIDAL
CAT

(4)

CONDITION : T IS SEMI-FLEXIBLE (DEF IN ABSTRACT)

THM : IF T IS SEMI-FLEXIBLE, THEN

$(A, TA \xrightarrow{\alpha} A)$ T -ALGEBRA & A EQUIVALENT TO X (BY R)
 THEN X ACQUIRES T -STRUCTURE ST R IS AN EQUIV. OF T -ALGS.

PROBLEM : "SEMI-FLEXIBILITY" IS HARD TO DEFINE AND VERY HARD TO VERIFY IN EXAMPLES.

SO SEEK CONDITION IN TERMS OF (L, E) THAT IMPLIES SEMI-FLEXIBILITY OF T .

ANSWER : NOT YET WRITTEN UP. STEVE LACH CONTRIBUTED CONDITION IS "NO EQUATIONS BETWEEN OBJECTS". THIS PRESUPPOSES PRECISE DEF OF "PRESENTATION".

~~STATE~~
 • FOR LAWVERE 2-THEORIES, PRESENTATION MEANS
 "FINITE-COTENSOR SKETCH" THAT IS SINGLE-SORTED.

• FOR FINITARY MONADS, MORE COMPLEX.

ONGOING WORK : WRITE IT UP!

