SET THEORY

$\exists x \exists y \; x \in y \in z \;\; \& \;\; z \leftrightarrow z'$

$\not\Rightarrow \exists x \exists y \; x \in y \in z'$

Ex: $z = \{\{\phi\}\} \qquad z' = \{\phi\}$

TOO EXPRESSIVE!

USUAL MATHEMATICS

IF $z \hookrightarrow z'$

PROPERTIES/STRUCTURE OF z

$\hookrightarrow$ PROPERTIES/STRUCTURE OF z'

CATEGORY THEORY

DESIGNED TO RESPECT $z \hookrightarrow z'$

Ex: TERMINAL OBJECT, PRODUCTS, LIMITS, CARTESIAN CLOSEDNESS, ...

(NON-Ex: STRICT MONOIDAL STRUCTURE.)

UNIVERSAL ALGEBRA

(E.g. GROUPS, MONOIDS, COMMUTATIVE MONOIDS, RINGS, R-MODULES, M-SETS)

$(\Sigma \quad E)$

Σ: OPERATIONS $\omega_1, \ldots, \omega_n$

E: EQUATIONS (UNIVERSALLY QUANTIFIED)

(Σ, E)-ALGEBRA $(A, op_1, \ldots, op_n)$

A: SET

op_i: $A \times \cdots \times A \to A$

IF $A \xrightarrow{R} X$, X ACQUIRES ALGEBRA STRUCTURE UNIQUELY SUCH THAT R IS AN ISOMORPHISM OF ALGEBRAS.

CATEGORY-THEORETIC FORMULATION

① LAWVERE THEORIES (CALLED ABSTRACT CLONES IN UNIV ALGEBRA) L

② FINITARY MONADS (SIMPLER DEF OF "MONAD", BUT "FINITARY" IS COMPLEX AND ESSENTIAL) T

IN BOTH FORMULATIONS, $A \xrightarrow{R} X \Rightarrow$ STRUCTURE TRANSFERS.

$(A, TA \xrightarrow{a} A)$ T-ALGEBRA $A \xrightarrow{R} X \Rightarrow X$ HAS UNIQUE T-ALG STRUCTURE S.T. R IS AN ISO OF ALGEBRAS.

WHAT ABOUT CATEGORIES WITH STRUCTURE?

- "UNIVERSAL ALGEBRA" ON CAT RATHER THAN SET
- EQUIV, LAWVERE 2-THEORIES $\mathbb{L}$
- EQUIV, 2-MONADS T ON 2-CATEGORY CAT

$(A, TA \xrightarrow{a} A)$ T-ALGEBRA $A \underset{X}{\overset{R}{\leftrightarrows}} X$ ISO OF CATS

$\Rightarrow X$ HAS UNIQUE T-STR S.T. R AN ISO OF T-ALGS.

BUT WHAT IF R IS AN EQUIVALENCE?

Exs: T_t = 2-MONAD S.T. T_t-ALG = CAT WITH TERMINAL OBJ

T_{FP} = " " " FINITE PRODUCTS

T_{FL} = " " "

T_{MON} = " " " T_{MON}-ALG = MONOIDAL CAT

NON-EX: T_{SMON} = " " " T_{SMON}-ALG = STRICT MONOIDAL CAT

CONDITION : T IS SEMI-FLEXIBLE (DEF IN ABSTRACT)

THM : IF T IS SEMI-FLEXIBLE, THEN
(A, $TA \xrightarrow{a} A$) T-ALGEBRA & A EQUIVALENT TO X (BY R)
THEN X ACQUIRES T-STRUCTURE ST R IS AN EQUIVALENCE
OF T-ALGS.

PROBLEM : "SEMI-FLEXIBILITY" IS HARD TO DEFINE
AND VERY HARD TO VERIFY IN EXAMPLES.
SO SEEK CONDITION IN TERMS OF (Ω, E)
THAT IMPLIES SEMI-FLEXIBILITY OF T.

ANSWER : NOT YET WRITTEN UP. STEVE LACK CONTRIBUTED.
CONDITION IS "NO EQUATIONS BETWEEN OBJECTS".
THIS PRESUPPOSES PRECISE DEF OF "PRESENTATION".
~~STARTET~~
- FOR LAWVERE 2-THEORIES, PRESENTATION MEANS
"FINITE-COPRODUCT SKETCH" THAT IS SINGLE-SORTED.
- FOR FINITARY MONADS, MORE COMPLEX.

ONGOING WORK : WRITE IT UP!