Games for Models, our first ideas

Glynn Winskel,

Adam Connolly, Sacha Huriot

Virtual Workshop on Resources & Coresources 26 March 2020

Aim :

To integrate a general theory of

gomes

with Ginite models.

Algeboras (Models)

Games on _____ Algebras

Resource - aware exploration of models

Games Computational

aspects: behaviour, resources, probability quantum

Traditional 2-party games, Player (+) vs. Opponent (-) as trees with winning conditions:



Traditional strategies, as subtrees, eg.



represents a deterministic strategy for Player



Or as maps

But games can be put in powallel, & even in the same gome moves can be n'dependent!

Another model ...

Definition

An event structure comprises $(E, \leq, \#)$, consisting of a set of events E

- partially ordered by \leqslant , the causal dependency relation, and

- a binary irreflexive symmetric relation, the conflict relation, which satisfy $\{e' \mid e' \leq e\}$ is finite and $e # e' \leq e'' \implies e # e''$.



Definition

The configurations, $C^{\infty}(E)$, of an event structure E consist of those subsets $x \subseteq E$ which are *Consistent:* $\forall e, e' \in x. \neg (e \# e')$ and *Down-closed:* $\forall e, e'. e' \leq e \in x \implies e' \in x.$

Often concentrate on the finite configurations C(E).

Maps of event structures

Definition

A map of event structures $f : E \to E'$ is a partial function on events $f : E \to E'$ such that

> for all $x \in C^{\infty}(E)$, $fx \in C(E')$ and if $e_1, e_2 \in x$ and $f(e_1) = f(e_2)$, then $e_1 = e_2$. (local injectivity)

Maps of event structures

Definition

A map of event structures $f : E \to E'$ is a partial function on events $f : E \to E'$ such that

for all
$$x \in C^{\infty}(E)$$
, $fx \in C(E')$ and
if $e_1, e_2 \in x$ and $f(e_1) = f(e_2)$, then $e_1 = e_2$. (local injectivity)

Maps preserve concurrency, and locally reflect causal dependency:

$$\forall x \in \mathcal{C}(E), e_1, e_2 \in x. \ f(e_1) \leqslant f(e_2) \implies e_1 \leqslant e_2.$$



Let 2 be a many-sorted relational signature. A \leq -game comprises an event structure (E, \leq, \times) • with polarity $pol: E \longrightarrow \{2+, -\}$ st. High =• sorted variables var: E --- > Var such that $e co e' \Rightarrow vor(e) \neq vor(e')$ · winning condition WC in free logic over 2, Var. X,BEVar Ela

Let 2 be a many-sorted relational signature. A \leq -game comprises an event structure (E, \leq, \times) • with polarity $pol: E \longrightarrow \{+, -\}$ st. $Hightharpool}$ • sorted variables var: E --- > Var such that $e \ co \ e' \Rightarrow \ var(e) \neq var(e')$ · winning condition WC in free logic over &, Var. WC: $E(\delta) \rightarrow \exists \beta . C(\alpha, \beta) \land S(\beta)$ L'Existence predicate \square_{χ} Ela Z,BeVar

Let 2 be a many-sorted relational signature. A \leq -game comprises an event structure (E, \leq, \times) • with polarity $pol: E \longrightarrow \{2+, -\}$ st. High =sorted variables 'var: E → Var such that
e co e' ⇒ var(e) ≠ var(e')
winning condition WC in free logic over ≤, Var. Let A be an algebra of signature \leq . An A-game comprises (A, E) with E a Z-game.

An A-strategy in an A-game (A, E) a total map of event structures Comprises • with an instantiation fr. \sum $\sim \frac{l_2}{4} \propto \frac{1}{2} \times \frac{1}{2} \times$ $inst: S \longrightarrow A$ m_{1} m_{1} m_{2} m_{3} m_{4} m_{5} m_{6} σ \ ↓ 0. • any enabling of \square_{α} in E& any sort-matching a in A has a unique enabling of |-__ Fa Fr \square_{a}^{a} in S.

É, Ér Er

 $WC: \alpha + \beta = \delta$

An A-strategy in an A-game (A, E) comprises a total map of event structures • with an instantiation fr. \sum $\sim l_2 \sim \lambda \chi$ $inst: S \longrightarrow A$ m_1 a b_1 a b_1 m_1 a b_2 b_3 m_1 a b_1 b_2 b_3 m_1 a b_2 b_3 b_3 m_1 • only extra immed. causal dependencies Est σ Λ σ • any enabling of \square_{α} in E& any sort-matching a in A has a unique enabling of |--- \square_{a}^{u} in S. Pa Pr It is winning if $x \neq WC_E$ for all $+-maximal x \in \mathbb{C}^{\infty}(S)$. $\dot{\Box}_{a}$ $\dot{\Xi}_{r}$ Ξ_{p} $WC: \alpha + \beta = \delta$

Strategies between A-games Dual $(\mathcal{A}, E, W_{\mathcal{E}})^{\perp} := (\mathcal{A}, E^{\perp}, \neg W_{\mathcal{E}})$ Pavallel composition $(A_{6}, E, WC_{E}) \otimes (B, F, WC_{F}) := (A \parallel B, E \parallel F, WC_{E} \vee WC_{F})$ "Function" space $(\mathcal{A}, E, \mathcal{W}_{E}) \rightarrow (\mathcal{B}, F, \mathcal{W}_{F}) := (\mathcal{A}, E, \mathcal{W}_{E})^{\dagger} \otimes (\mathcal{B}, F, \mathcal{W}_{F})$ $= (\mathcal{A} || \mathcal{B}, \mathcal{E}^{\perp} || \mathcal{F}, \mathcal{W} \mathcal{C}_{\mathcal{E}} \to \mathcal{W} \mathcal{C}_{\mathcal{F}})$ A (winning) A-strategy from (A,E) to (B,F) is a (winning) A-strategy in (A,E)+> (B,F). (Winning) strategies $\forall: (A, E) + (B, F), \forall: (B, F) + (E, G) compose:$ play them off against eachother over (B, F).

The copyest A-shategy - A has provalues t.f.



copycat: (A,E) +> (A,E) identity A-strategy

'd2



The copyest A-shategy - A has provalues t, f. ×B2 p copycat factors Emongh "symbolic" -B2 dz copycat game A 22 -B2 א.

Let A, B be E-algebras.

Can realise

honomorphisms A -> B and

as

Game (A) +> Game (B) and

Res_k (Game (A)) +> Game (B).

(1) via "access levels" to make A-games of imperfect nifo. Two ways: (2) via winning strategies (A, G, WC) +> (B, G, WC) which aproject" to symbolic copyet G+>G.

Game (,



Game (A) + Game (B)









 $WC_A \longrightarrow WC_B$

Its copycat game:



Ehren feucht - Traisse

the copycat game g:

isomorphismigane got from

Game (B)

S S RV S

in Rv

2 Duj

two -way

WC_B:

 $\bigwedge_{n R^{\vee}} \mathbb{E}(n R^{\vee}) \longrightarrow \neg R(\beta_{v_1, \cdots, \beta_{v_n}})$

 $\bigwedge_{ij} E(ij) \longrightarrow Bi = Bj$

 $\bigwedge_{ij} \mathbb{E}(nij) \longrightarrow \neg \beta^{i} = \beta^{j}$

tentative v. partial Games on algebras picEure: fibre over $(A, E, W_{E}) \rightarrow (B, F, W_{E})$ Algebras with gueries? expansion 'Symbolic'games & strategies familier of relations $(\mathcal{A}, \mathcal{W}_{\mathsf{E}}) \xrightarrow{\mathcal{F}} (\mathcal{B}, \mathcal{W}_{\mathsf{F}})$ Concurrent games with algebra onevents E+>F E - - > copycat Adjunctions? Algebras Fibrations ? Resp